Geometric realisation of Connes spectral triples for algebras with central bases

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Outline

NCRG statement for quantum gravity

1. We will not assume that the spacetime is continuum at the plank scale.
2. Instead propose that it is more e ectively described by a noncommutative coordinate algebra.
3. And in the limit classical RG is recovered.

[1] E.J. Beggs and S. Majid, Quantum Riemannian Geometry, Grundlehren der mathematischen Wissenschaften, Vol. 355, Springer (2020) 809pp

Basic NCRG formalism

We work with A a unital algebra, typically a -algebra over C, in the role of `coordinate algebra'.

1. Di erentials are formally introduced as a bimodule ^{1} of 1-forms equipped with a map d : A ! ^{1} obeying the Leibniz rule d(ab) = (da)b + adb.
2. Assume this extends to an exterior algebra (; d) with $d^2 = 0$ and d obeying the graded-Leibniz rule and ^(g) = 0.
3. A quantum metric is g 2 ^{1} $_A$ ^{1} and a bimodule map inverse (;) : ^{1} $_A$ ^{1} ! A.
4. A bimodule connection on ^{1} is r : ^{1} ! ^{1} $_A$ ^{1} obeying:
   1. r (a:!) = a:r ! + da !,
   2. r (!: a) = (r !):a + (! da)

   with a unique `generalised braiding' bimodule map : ^{1} $_A$ ^{1} ! ^{1} $_A$ ^{1}.

General axiomatic motivation

In recent years, the quantum Riemannian geometry was extended to a systematic theory including the QLC and further structure as:

1. 'spinor' bimodule S equipped with a bimodule connection $_S$
2. A 'Cli ord action' $. : {}^1 {}_A S \,!\, S$.
3. Leading to a quantum-geometric Dirac operator $D = . \circ r_S$.
4. And a inner product used to complete S to a Hilbert Space.

Motivation for central bases

We say a basis e^i is central if is a grassmann algebra: $e^i e^j + e^j e^i = 0$.

We will think in central bases if A has trivial centre, ^{1} has a central basis $\{s^i\}$ and S has a central basis $\{e^\alpha\}$.

The central bases assumtion resumes into the set of 1-forms are self adjoint. The metric translates to the matrix g_{ij} of metric coefficient in the basis being hermitian. If is the flip map then the -preserving condition on translates to the Christoffel symbols in the basis being real.

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Some results and examples

Theorem

Up to a phase in the 2D spinor bundle case, J can be obtained with $r > 0$ and $z \in \mathbb{C}$ as either:

$$(1): \quad J = \begin{pmatrix} z & r \\ \frac{|z|^2}{r} & z \end{pmatrix} \; ; \quad (2): \quad J = \begin{pmatrix} 1 & \frac{1}{|z|^2} z \\ z & \frac{z}{z} \end{pmatrix}$$

or its transpose. The $=$ 1 case of (2) needs $z \neq 0$ and up to a

Some results and examples

Noncommutative Torus

the only geometrically realised Dirac operator for the standard Euclidean metric and a WQLC are

$$D(\quad e) = (@ \quad s^i).e + \quad d_i s^i.e = {}^i \quad ((@ + d_i) \quad)e : \quad (1)$$

. With Hilbert space, the state $\int_R u^m v^n = \quad _{m;0} \quad _{n;0}$